Computational model of unsteady hydromechanics of large amplitude Gerstner waves

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Abstract. Numerical experiments in ship hydromechanics involve non-stationary interaction of a ship hull and wavy surface that include formation of vortices, surfaces of jet discontinuities, and discontinuities in fluid under influence of negative pressure. These physical phenomena occur not only near ship hull, but also at a distance where waves break as a result of interference of sea waves and waves reflected from the hull. In the study reported here we simulate wave breaking and reflection near the ship hull. We use explicit numerical schemes to simulate propagation of large-amplitude sea waves and their transformation after the impact with a ship. The problem reduces to determining wave kinematics on a moving boundary of a ship hull and a free boundary of a computational domain. We build a grid of large particles having a form of a parallelepiped, and in wave equation in place of velocity field we integrate streams of fluid represented by functions as smooth as wavy surface elevation field. We assume that within boundaries of computational domain waves do not disperse, i.e. their length and period stays the same. Under this assumption we simulate trochoidal Gerstner waves of a particular period. Wavy surface boundary have to satisfy Bernoulli equation: pressure on the surface of the wave becomes non-constant, fluid particles drift in the upper layers of a fluid in the direction of wave propagation, and vortices form as a result. The drift is simulated by changing curvatures of particles trajectories based on the instantaneous change of wavy surface elevation. This approach allows to simulate wave breaking and reflection near ship hull. The goal of the research is to develop a new method of taking wave reflection into account in ship motion simulations as an alternative to the classic method that uses added masses.

1 Introduction

Ship and sea wavy surface motion do not have an abundance of geometric forms and physical phenomena. Waves formation due to ship motion and interaction between ocean and atmosphere are governed by continuity condition for heavy fluid and the law of conservation of energy.

Strict theoretical solution (and in fact the only solution) for large-amplitude wind waves on the surface of heavy fluid was obtained in 1802 by Franz Josef von Gerstner — a professor from a university in Prague [1]. Generic trochoidal wave mathematical model has large dispersion [2], the dependence of the speed of wave propagation on their length and period. As a result

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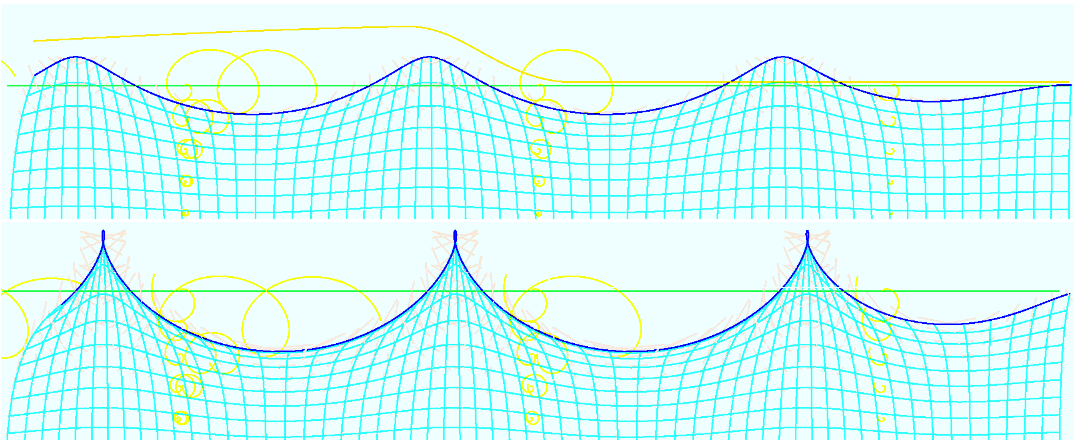

Figure 1. Analytic solutions: progressive Gerstner wave (top), a wave with critical height — a standing wave (bottom).

- wave energy propagation speed becomes half the visible phase velocity of wave crests,
- wave front constantly changes its phase, and
- wave are quantised into packets and wave transformation and propagation phenomena become non-stationary.

2 Computational model of intense sea waves

Gerstner wave (fig. 1) is a cycloid, fluid particle trajectory radius $r_W = 1.134\lambda_W h_W / 4\pi$ [m] of which is fixed relative to flat wavy surface level z_W, hence z-coordinates of the crest and trough are the same. Here λ_W is the wave length, h_W — relative wave height defined on the interval [0..1] with $h_W = 1$ being the maximum wave height for which the crest does not break (fig. 1). Vertical displacement of a fluid particle is given by

$$\zeta_Z = r_W \cos x_W \exp\left(-2\pi z_W / \lambda_W\right) \qquad [\mathrm{m}]\,.$$

Horizontal displacement of the same fluid particle with respect to its initial position for progressive wave is given by analogous equation, but with a shift by one fourth of the phase:

$$\zeta_X = -r_W \sin x_W \exp\left(-2\pi z_W / \lambda_W\right) \qquad [\mathrm{m}]\,.$$

Critical wave height of Gerstner waves (fig. 1) gives the correct ratio of wave height to wave length, but 60 degree slope limit for standing wave with steepness $\approx 1/4$ as well as 30 degree slope limit for progressive (traveling) wave with steepness $\approx 1/7$ are not correctly captured by the model.

State of the art mathematical and computational models do not simulate wave groups that are integral part of ocean wavy surface motion. Our model, which is a modified version of Gertner wave, includes wave groups. They are described as a dependency between fluid particle trajectory radius and instantaneous displacement of the particle with respect to calm sea level (fig. 2).

We write adjusted radius as ${}^A R_W = {}^A K r_W \left(\cos x_W - 1\right)$, where ${}^A K = [1, 0..\sqrt{2}]$ is radius coefficient that makes wave crests cnoidal and raises mean sea level. We choose ${}^A K$ to be slightly less than 1

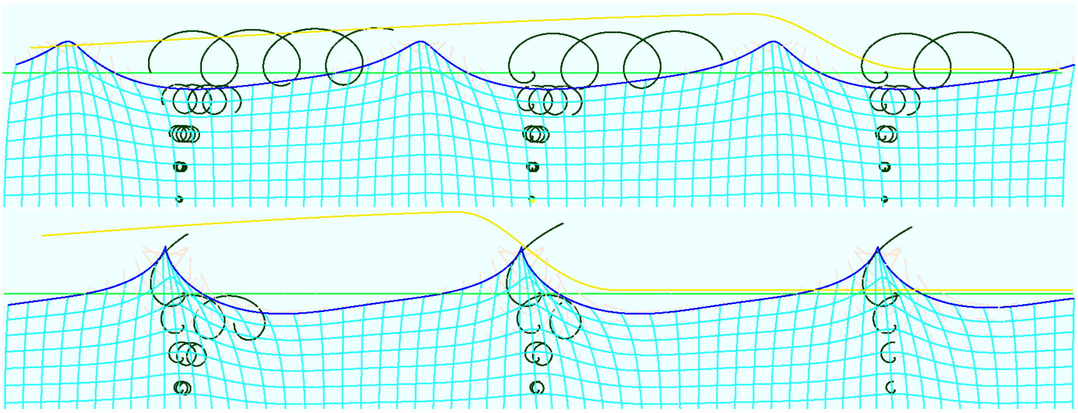

Figure 2. Simulation result: regular trochoidal waves with vertical displacement of sea level and wind stress on the wave sea surface. Propagating wind waves (top), extremely high wind waves (bottom).

to reduce the effect of gusty winds on the wave form and prevent forming of cycloidal loops in wave crests, that appear for waves with overly large amplitude, that may occur as a result of the interference with waves heading from the opposite direction.

The pressure on windward slope of the wave is smaller, because wind slides on the surface of the wave at a high speed and makes the slope more flat, while on the leeward slope of the wave wind speed drops significantly or even goes to nought and creates vorticity.

The coefficient of wind stress ${}^{W}K$ (the parameter that was shifted by one fourth of the phase) determines the assymmetry of steepness of windward and leeward slopes of the wave (fig. 2): ${}^{W}R_W = {}^{W}K r_W (\sin x_W - 1)$, where ${}^{W}K = [0..1]$. The coefficient is close to unity for fresh wind waves and close to nought for swell. For two-dimensional sea surface ${}^{W}K$ is used in dot product between wind and wave direction vectors:

$$\zeta_Z = r_W \cos x_W \exp\left(2\pi\left[-z_W + r_W^A K(\cos x_W - 1) + r_W^W K \sin x_W\right]/\lambda_W\right)$$
$$\zeta_X = -r_W \sin x_W \exp\left(2\pi\left[-z_W + r_W^A K(\cos x_W - 1) + r_W^W K \sin x_W\right]/\lambda_W\right).$$

Then energy conservation is defined by Bernoulli's principle

$$\frac{\rho V^2}{2} + \rho g \zeta_W = \text{const}, \qquad \left[\text{N/m}^2\right]$$

where particle velocity V contributes the most to balancing the pressure $\rho g \zeta_W$ on the wavy surface down to nought for breaking waves.

3 Trochoidal wave groups

In our modified model we simulate two wavy surfaces simultaneously: one for regular waves with normal length and one for waves with nine times higher length, that propagate under the same laws but with two times less speed (fig. 3). The product of these surfaces allows to simulate wave groups.

On the first entry the profile of the long wave is given by specific smoothing function, the form of which is close to phase wave profile. This function defines continuous change of wave front phase, which is needed to simulate waves produced by the ship.

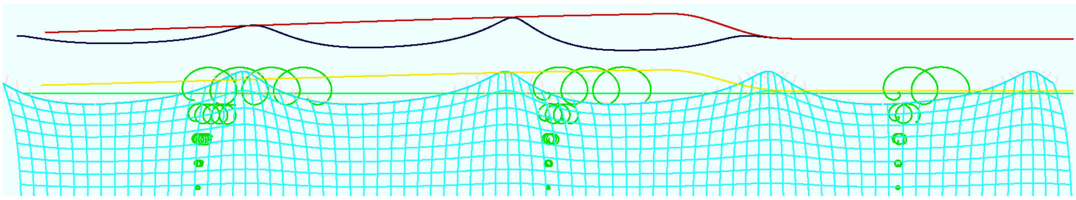

Figure 3. Trochoidal wave groups.

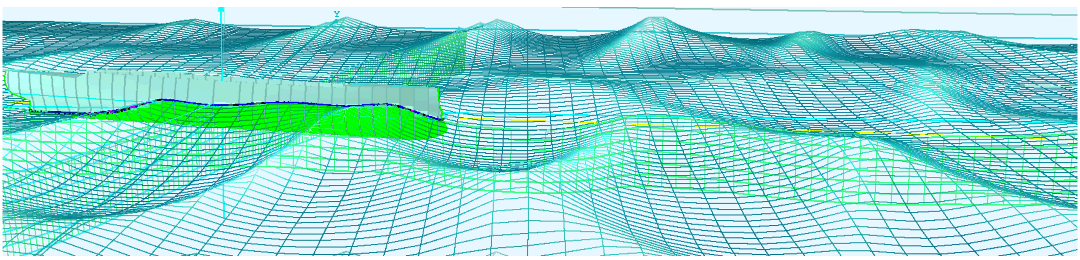

Figure 4. Large-amplitude trochoidal waves.

There is also a simpler approach to simulate wave groups: a superposition of regular waves with slightly different periods propagating in the opposite directions. Interference of waves of comparable lengths produces beats, in which nineth waves has double height and are standing waves. This approach generally gives satisfactory wavy surface, but does not work for waves produced by the ship, because they have complex wave front.

4 Direct numerical simulation of sea waves

We use explicit numerical scheme to simulate sea wavy surface that satisfies continuity equation; we call it direct numerical simulation (fig. 4). We use the following definitions for three sea wave systems, that are used in the scheme.

- Fresh wind waves have a period of 6-8 seconds near the shore and up to 10-12 seconds in the ocean. The height of the wave is close to critical, that typically corresponds to 6 on Beaufort scale with wave crests greater than 5-6 metres.
- Fresh swell waves skew from mean wind direction by $\approx$ 30 degrees. When the storm in northern hemisphere increases, wind direction goes counterclockwise and vice versa, i.e. the swell is always present in the ocean. Swell waves are comparable to wind waves: their height is two times smaller than critical wave height, and their length is 1.5-2 times greater.
- Old swell waves are long waves that come from higher latitudes. Their height is two times smaller than than of wind waves and fresh swell, their length is two times greater, and their direction is close to meridional (i.e. south in northern latitudes and vice versa[1]).

These waves may add up in unfavourable way to a wave with the height of 13-15 metres, however, in real world mean wave height will be 8-10 metres. Wave groups have nineth wave with double height, breaking crest and wave slope greater than 45 degrees.

[1] The wind blows into the compass rose, the waves propagate in the direction of the rose.

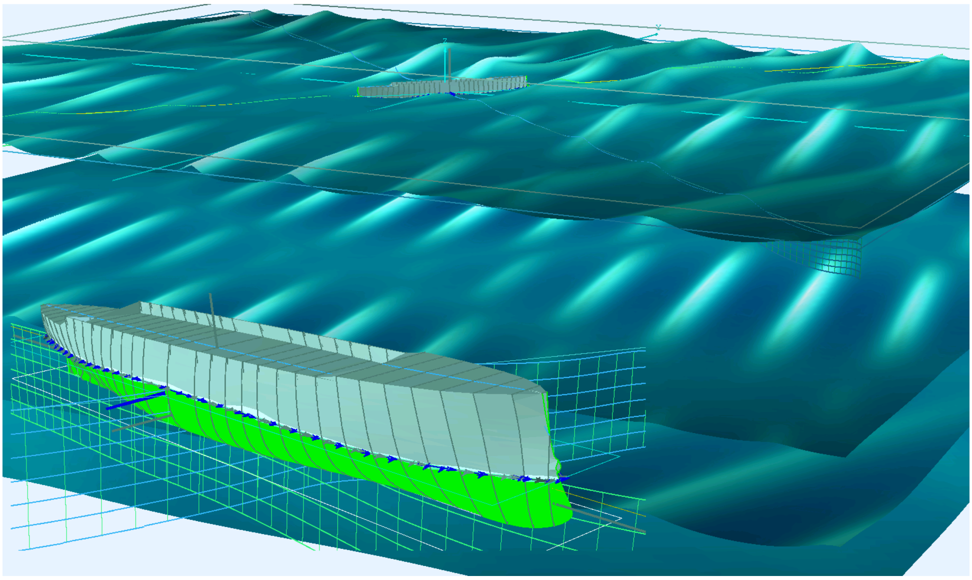

Figure 5. In the course of the simulation we visualise all three wave systems and create a view of ship hull dynamics and sea wave profiles in a different convenient scale.

Oceanographers use well-established solutions [3] for regular progressive waves of arbitrary shape. Using trochoidal waves as a source, we fix wave periods and speeds in time to satisfy continuity equation and energy conservation law. We simulate all three wave systems (described above) simultaneously and indepedently (fig. 5) and add individual wavy surfaces together to produce the resulting wavy surface.

5 Conclusion

We use explicit numerical schemes to simulate modified version of Gerstner waves. We simulate particle drift in the upper fluid layers by changing the curvature of the trajectory depending on the instantaneous change of wavy surface elevation. Our model is nonstationary, hence ship motions can also be nonstationary. Computational power of a desktop computer is enough for performing such simulations in real-time, and these types of simulations can even be performed on the board of the ship to chose optimal and efficient mode of ship operation.

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